Semi-analytical LLG Solver Using Effective Field Time Derivative

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I. Abstract

For fixed H, the Landau-Lifshitz equation

$$\frac{dm}{dt} = \frac{\gamma}{1+\alpha^2} m \times H - \frac{\alpha\gamma}{1+\alpha^2} m \times H \times m, \tag{1}$$

has analytic solution, expressed in spherical coordinates,

$$\phi(t) = |\gamma H| t, \tag{2}$$
$$\theta(t) = 2\tan^{-1}(\tan(\theta(0)/2)\exp(-|\alpha\gamma H|t)). \tag{3}$$

Repeated application over short enough time intervals leads to a semi-analytic solver [1] for time-varying H. Any field adjustment $\tilde{H} = H + \lambda m$ creates no change in torque

$$m \times \tilde{H} = m \times H; \tag{4}$$

thus, no change in the solution of (1). Selecting field adjustments to support longer semi-analytic time intervals can improve efficiency [2]. The simple adjustment $\tilde{H} = H - (H \cdot m)m = m \times H \times m$ can be computed from available data without increasing storage or computational requirements. These are explicit, norm-preserving solution methods with clear extensions to predictor-corrector schemes.

Here we extend this approach to consider polynomial time variations of H, constrained by dH/dt. Computing dH/dt costs the same as computing H. Trajectories of m corresponding to time-varying H are computed numerically building on (2) and (3). A predictor-corrector scheme assumes linear H to predict m at the end of the time step, computes H there, and then computes corrected values of m assuming a quadratic H.

Fig. 1 graphs the accuracy of these methods as a function of time step with comparison to conventional Runge-Kutta solvers. A 240 nm ×240 nm ×12 nm Permalloy plate is simulated using 4 nm cubic cells. A field $(1, 2, 10)$ mT is applied to an initial uniform magnetization slightly off the x axis, and 200 ps of response are simulated. Error is computed as the difference of the final average m_y from that computed by a baseline converged solution. The solver using dH/dt is third order in the time step. It is also able to take longer time steps on the same problem than both the semi-analytic method built on $\tilde{H} = m \times H \times m$ and a second order Runge Kutta solver before experiencing the total loss of accuracy indicating numerical instability. Longer time steps while maintaining stability and accuracy indicate a more efficient calculation. Fig. 2 displays the same data as Fig. 1, but scales the horizontal axis by the number of field evaluations. This offers a clearer comparison of the tradeoff between accuracy and stability and computational effort.

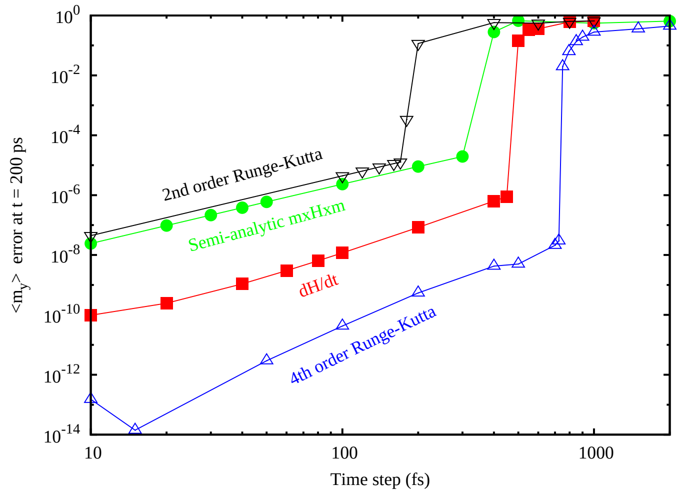

Fig. 1. Error performance as function of time step.

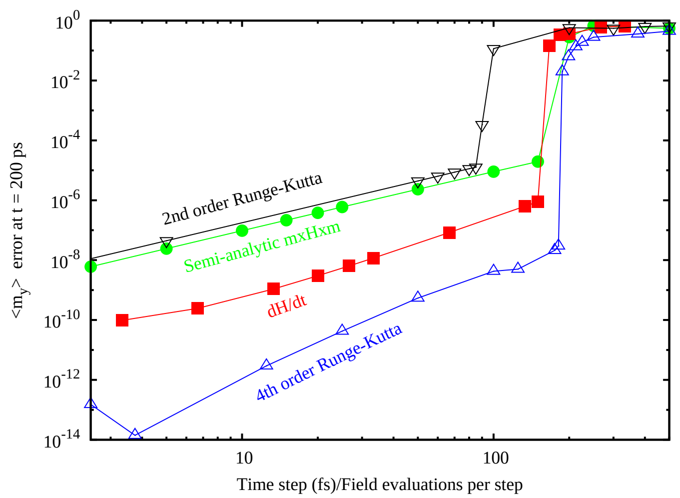

Fig. 2. Error performance as function of time step and field evaluations.

References

[1] Ben Van de Wiele, Femke Olyslager, and Luc Dupré, "Fast semi-analytical time integration schemes for the landau-lifshitz equation", *IEEE Trans. Magn.*, vol. 43, no. 6, pp. 2917–2919, June 2007.

[2] Donald Gene Porter and Michael J. Donahue, "Precession axis modification to a semianalytical landau-lifshitz solution technique", *J. Appl. Phys.*, vol. 103, no. 7, pp. D920, February 2008.

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