# Modeling of noise passage through complex hysteretic systems

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## I. INTRODUCTION

The major role played by noise and fluctuations at the nanoscale level has raised a large amount of interest into the area of nonlinear systems driven by stochastic inputs. A special focus is on magnetic recording technology, where thermal effects pose fundamentals limits for further improvements in magnetic storage density. However, the systematic approach to the stochastic aspects of nonlinear systems with memory has been rather limited due to the lack of appropriate tools needed for the analysis of complex non-Markovian processes represented by the system outputs. [1]

Recent studies have shown that relatively simple hysteresis models can describe a large range of noise induced phenomena from the occurrence of 1/f noise and thermal relaxations [2, 3] to coherence and stochastic resonances [4, 5]. Closed form solutions for the spectra of the output processes have been derived for *symmetric* Preisach systems with stochastic input [6] by combining the classical decomposition into symmetric rectangular operators with the recently developed theory of stochastic processes on graphs [7].

In this article, the stochastic analysis is extended to other phenomenological models of hysteresis, such as the Jiles-Atherton model (JAM), the Hodgon model (HM), the Energetic model (EM), and the *general* Preisach model (PM). A pertinent comparison of the results obtained by using these integral, differential, and transcendental models of hysteresis driven by noisy inputs is presented and their general stochastic features are discussed. It is also shown that the spectral analysis is a powerful characterization tool that can be used to design filters based on nonlinear hysteresis.

## II. MODELING AND COMPUTATIONAL RESULTS

Our approach to the stochastic analysis of hysteretic systems involves a diffusion process input described by the Ito stochastic differential equation:

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t, \tag{1}$$

where $W_t$ is the Wiener process, while $b$ and $\sigma$ are given functions of the current process state $x$ that satisfy local Lipshitz and linear growth conditions [8]. The input-output relation of the hysteretic system was subsequently described by JAM, HM, EM, and PM. In order to compute the stochastic characteristics, numerous Monte-Carlo simulations were performed for each model and the final results were averaged out. The computational results obtained for rectangular loops and *symmetric* Preisach systems were consistent to the semi-analytical results presented in [1, 6], which provided a successful test for our general approach.

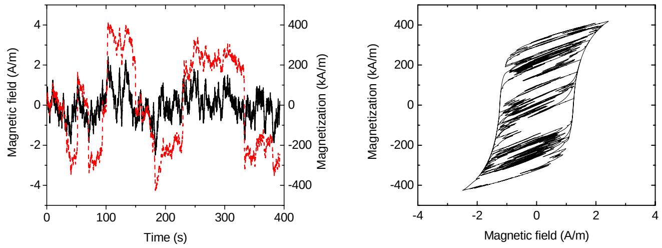

Fig. 1. Realization of input (black) and output (red) processes for EM model (left) and the corresponding input-output diagram (right).

In Fig. 1 a realization of the Ornstein-Uhlenbeck (OU) input process and the corresponding output is presented along with the input-output diagram for the case of EM, while in Fig. 2, the output power spectral densities are plotted for various values of the OU input diffusion coefficient. Although the input spectra are Lorentzian (flat for low-frequencies (LF) and $1/f^2$ decay for high-frequencies (HF)), the output spectra feature various $1/f^{\alpha}$ decays for HF and non-flat behavior at LF.

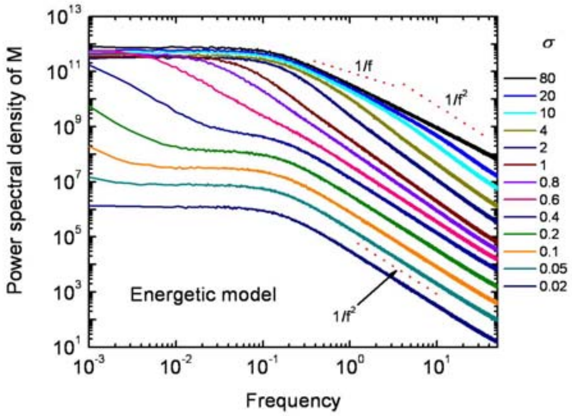

Fig. 2. Spectral densities for the Energetic model for selected values of σ.

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