A simple hysteretic rainfall-runoff model

D. P. Flynn, J. P. O'Kane

I. Introduction

Multiphase porous media such as soils are known to exhibit hysteresis. In particular, unsaturated soils exhibit rate-independent hysteresis between the moisture content and the capillary pressure. To date the Wedge model which is a special case of the Preisach model, has been successful in modelling this relationship, where it was used to fit a series of soil data with better results than previous models[1]. Subsequently the Wedge model was used to develop a new class ODEs where the Preisach operator was on the left-hand-side of the ODE [2], [3]. Also developed was a hysteretic version of the linear reservoir, which is a model for water flow in large catchments[4], [5], we shall refer to this model as the Preisach reservoir.

We will now show how these previous models i.e. the hysteretic ODE based on Darcy's law and the Preisach reservoir can be combined to produce a hysteretic rainfall-runoff model.

II. Model

A simple conceptual rainfall-runoff model is shown in Figure 1. It is composed of three subsystems: one representing surface water, the second representing groundwater storage and outflow, and the third representing the unsaturated soil moisture zone.

Figure 1 also charts a part of the hydrological process, which initially begins with rainfall entering the soil. If enough water is present in the soil, it will subsequently drain into the groundwater reservoir. Sometimes the rainfall rate can exceed the maximum infiltration rate, and this excess water becomes the overland flow. Finally the overland flow and groundwater will meet at some point and this combined output becomes the runoff.

One of the simplest models as suggest by [6] had a single linear reservoir for the groundwater reservoir, and two linear reservoirs in series for the overland flow. However, in our work, we used Preisach reservoirs instead of these three linear reservoirs, and we used a hysteretic ODE based on Darcy's law to represent the soil component.

Finally we present a comparison of the qualitative behaviour of the hysteretic rainfall-runoff with its non-hysteretic counterpart.

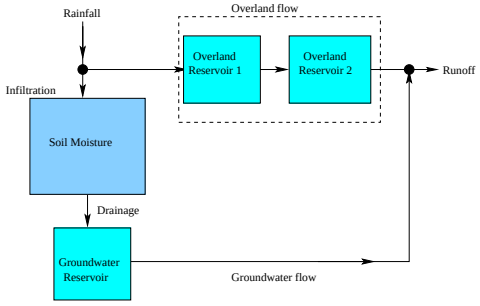

Fig. 1. Rainfall-runoff model with hysteresis.

References

[1] D. Flynn, H. McNamara, P. O'Kane, and A. Pokrovskii, *Science of Hysteresis: Application of the Preisach model to Soil-Moisture Hysteresis*. Academic Press, 2005, vol. 3, no. ISBN: 0-12-480874-3, ch. 7, pp. 689–744.

[2] D. Flynn and O. Rasskazov, "On the integration of an ODE involving the derivative of a Preisach nonlinearity," *Journal of Physics: Conference Series*, vol. 22, pp. 43–55, 2005. [Online]. Available: http://stacks.iop.org/1742-6596/22/43

[3] D. Flynn, J. P. O'Kane, and A. Zhezherun, "Numerical solution of odes involving the derivative of a preisach operator and with discontinuous rhs," *Journal of Physics: Conference Series*, vol. 55, pp. 63–73, 2006. [Online]. Available: http://stacks.iop.org/1742-6596/55/63

[4] J. P. O'Kane, "The hysteretic linear reservoir–a new preisach model," *Physica B: Condensed Matter*, vol. 372, no. 1-2, pp. 388–392, February 2006.

[5] J. P. O'Kane and D. Flynn, "Thresholds, switches and hysteresis in hydrology from the pedon to the catchment scale: a non-linear systems theory," *Hydrol. Earth Syst. Sci.*, vol. 11, no. 1, pp. 443–459, 17 Jan. 2007.

[6] J. Dooge and J. O'Kane, *Deterministic Methods in Systems Hydrology*. A.A. Balkema Publishers, Lisse (NL), 2003.

D. P. Flynn is with the Department of Applied Mathematics, University College Cork (UCC), Ireland. E-mail: d.flynn@ucc.ie

J.P. O'Kane is with the Department of Civil & Environmental Engineering, University College Cork (UCC), Ireland. E-mail: p.okane@ucc.ie