Analysis of a unit magnetic particle via the DPC Model

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I. INTRODUCTION

Recently a generalized rate-independent vector model of static magnetic hysteresis was introduced, called DPC (Della Torre, Pinzaglia and Cardelli) model. This model is centered on the rule of a phenomenological vector hysteresis operator (hysteron), defined in the ***H***- space. This operator generates a vector magnetic field that has unit value magnetization. The vector hysteresis operator is defined by means of a convex surface, called critical surface, which is the locus of the values of ***H*** where there is discontinuity of the magnetization and a Barkhausen-like jump. For further details about the DPC model refer to [1] and [2].

II. THE DPC MODEL FOR AN UNIT MAGNETIC PARTICLE

The variation of the density of the magnetic energy of the particle for any cycle of ***H*** outside the critical surface of the hysteron must be zero, because the only possible loss of energy defined in the DPC model occurs when the critical surface is crossed from inside to outside. It follows that the vector magnetic field created by the magnetic particle must be conservative for any cycle of ***H*** out of the critical surface of the hysteron. Therefore the critical surface of the hysteron must be an equipotential surface for the vector field **M**(***H***), and the magnetization for values of ***H*** out of the critical surface must be directed along the lines of force of the vector field.

For a sake of convenience and simplicity in the mathematical formulation, here it is discussed the 2-D case with $\boldsymbol{H_I} = 0$, but the results can be easily extended to the 3D case with an arbitrary translation $\boldsymbol{H_I}$ from the origin. It is now convenient to express magnetization and applied magnetic field as a function of the coordinate parameters (u, v). The conditions that $|\boldsymbol{M}| = 1$ leads to

$$M_x = \cos v\ ,\ M_y = \sin v\ . \qquad (1)$$

Now, if the vector field **M**(***H***) is conservative outside the critical surface from the classic theory of vector fields, with suitable mathematical manipulations

$$H_x = u\cos v + \phi(v)\ ,\ H_y = u\sin v + \psi(v) \qquad (2)$$

where

$$\frac{\partial\phi}{\partial v}\cos v + \frac{\partial\psi}{\partial v}\sin v = 0\ . \qquad (3)$$

The analysis of (1) (2) and (3) proves that:
- the lines of force of the vector field created by the unit magnetic particle are straight lines, and the value of v represents the slope of the line of force and the direction of the unit magnetization;
- the magnetic scalar potential of the vector field generated by the unit magnetic particle is equal to u plus an arbitrary real constant, and for $u = const$ the equations (3) and (4) describe the family of equipotential surfaces of the vector field ***M***(***H***);
- if the domain of the conservative field must be the entire ***H***-plane, then the equipotential curves are circles;
- in general these solutions describe closed equipotential curves that approach circles as the applied field goes to the infinity. In the extended version of the paper it will be shown that, if we exclude a region around the origin from the entire ***H***-plane, other solutions different from circles are possible. One of them leads to results formally identical to those derived by a milestone paper of Stoner and Wohlfarth [3]. Fig.1 shows The cycles $M_x(H_x)$ for an alternative applied field directed along different directions from 0° to 75° respect to the hard axis of the magnetic particle described with $u = 0$.

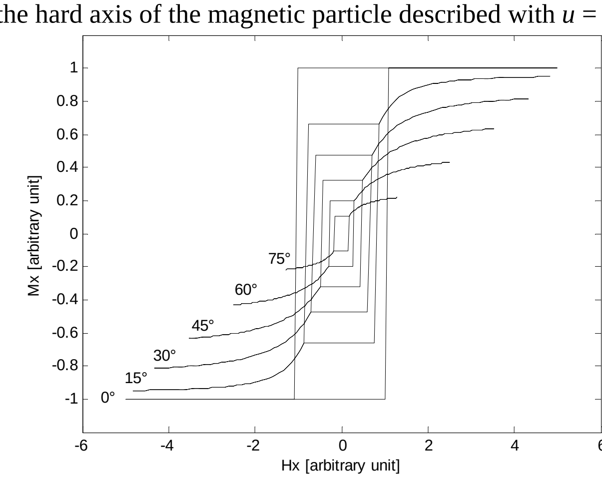

Fig.1 – Hysteresis loops generated by an unit magnetic particle

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REFERENCES

[1] E. Cardelli, E. Della Torre, A. Faba, "Properties of a class of vector hysterons", Journal of Applied Physics, 103, 07D927 (2008).

[2] E. Cardelli, E. Della Torre, and A. Faba, "Numerical Implementation of the DPC Model", IEEE Transaction on Magnetics, VOL. 45, NO. 3, March 2009.

[3] E.C Stoner, E.P. Wohlfarth " A Mechanism of Magnetic Hysteresis in heterogeneous alloys", Philos. Trans. R. Soc. London A240 pp. 599-642, 1948.