Analytical description of quasi-random magnetization relaxation to equilibrium

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Magnetization relaxation processes appear in many situations of fundamental and technological interest, such as the operation of spin valve elements, precessional switching of the magnetization or switching induced by spin polarized currents.

Due to the fact that the damping constant in the Landau-Lifshitz (LL) equation is a small quantity, two widely different time scales are present in the relaxation process: a fast one, over which precessional (conservative) motion of the magnetization occurs, and a slow one, over which the free energy of the system relaxes to its final equilibrium value.

By carrying out averaging of the LL equation with respect to the fast time scale, the differential equation which governs the slow decay of the system free energy is derived. The derivation is based on the knowledge of the trajectories of the 'unperturbed' (undamped) conservative motion which can be analytically determined.

It turns out that the above averaging technique (and the mentioned separation of time scales) breaks up when relaxation leads magnetization to cross the separatrices (trajectories ending or starting at saddle equilibria) of the unperturbed conservative motion.

In this work, we discuss how it is possible to extend the averaging technique in presence of this separatrix crossing [1]. Interestingly, due to the small damping constant, this phenomenon gives rise to a quasi-random behaviour in magnetization relaxations [2].

This means that, in presence of multiple stable equilibria, very small perturbations of the initial condition may induce magnetization to relax to one or another stable equilibrium

This issue can be explained by looking at the phase portrait of the dynamics, sketched in fig. 1. Initial conditions in the shaded region enclosed by the separatrices will end in the stable equilibrium s_1, whereas initial conditions in the white region will end in the stable equilibrium s_2. The aforementioned regions are called basins of attractions of the equilibria s_1 and s_2. It is worth to notice that such basins of attractions are tightly riddled and that their entanglement is more and more pronounced as far as the damping is smaller.

In this situation, it is clear that small perturbations in the initial conditions may lead to different magnetization relaxations, which in the limit of small damping can be described in probabilistic terms.

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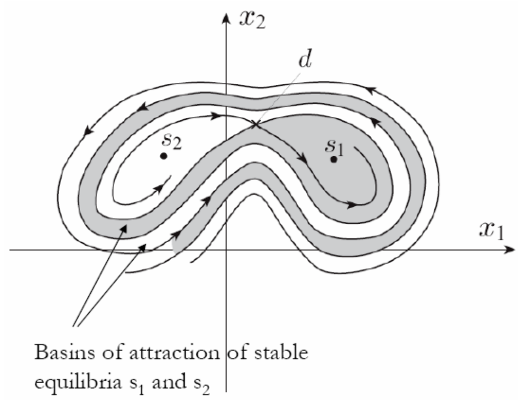

Fig.1. Sketch of the phase portrait of LL dynamics. The shaded (white) region enclosed by the separatrices represents initial conditions ending in s_1 (s_2).

In this work, we derive analytical formulas for the probabilities that magnetization will eventually relax in one of the stable equilibria. These formulas enable one to 'glue' the solutions of the averaged energy equation across separatrices and thus to obtain a solution valid, albeit only in the probabilistic sense, in the whole state space.

In fig. 2 we report the analytically computed probabilities for the case of an ellipsoidal particle with long x axis, initially magnetized along m_x=-1, and subject to a DC field applied along the positive x axis. The field amplitude is smaller than the coercivity of the particle. One can clearly see that in zero field both equilibria are equiprobable and when the field is equal to the coercivity, m_x=-1 becomes unstable.

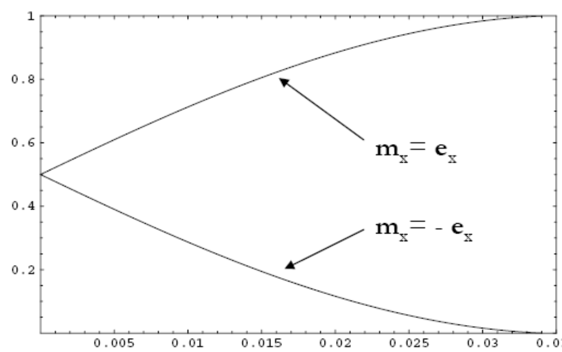

Fig.2. Probabilities that magnetization will eventually relax in one of the stable equilibria as function of applied field. The coercivity is 0.034 (dimensionless units).

References

[1] N.R. Lebovitz, A.I. Pesci, SIAM J. on Appl. Math **55**, 1117 (1995)
[2] G. Bertotti, I.D. Mayergoyz, C. Serpico, M. d'Aquino, IEEE Trans. On Magn. **39**, 2501(2003)