Numerical solutions to Fokker-Planck equation for magnetic nanoparticles

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I. Introduction

The study of thermal fluctuations in magnetization dynamics is particularly relevant to long-term stability of information recorded in the magnetic supports. From the theoretical point of view, thermal fluctuations in magnetization dynamics are usually taken into account by augmenting the effective field in the Landau-Lifshitz-Gilbert equation with an isotropic Gaussian white noise random field [1]. If the magnetic particle is sufficiently small, it can be assumed that the magnetization is spatially uniform and its direction fluctuates because of thermal agitation. In this framework, a complete analysis of this phenomenon can be carried out through a probabilistic approach by solving the associated Fokker-Planck (FP) equation that provides the transient evolution of the probability density [1]. In this paper we present two different methods to achieve relevant numerical results. The first method is an algebraic approach that uses integral variables defined on a cells' complex [2]. The two variables that completely describe the problem are the probability P defined in each cell and the flowed probability Q defined on the boundary of each cell. The use of such direct algebraic description is interesting because it is no longer necessary to carry out discretization of the FP equation. The second method is a spectral collocation scheme that interpolates the probability density in the parameters space using Lagrange polynomials on a grid of points [3].

II. Numerical results

In order to compare the two numerical formulations described above, we considered the analysis of thermal fluctuations for a small magnetic nanoparticle. The material parameters are: $M_s = 8 \cdot 10^5$ A/m, negligible magnetocrystalline anisotropy, damping constant $\alpha = 0.02$. We assume temperature $T = 300$ K and no external field applied. The chosen magnetic particle has spheroidal shape and the ratio of the major axis to the minor one is 3. We considered three different volumes of the particle: $V_1 = 5.8178 \cdot 10^{-26}\mathrm{m}^3$, $V_2 = 3/2 \cdot V_1$, and $V_3 = 2 \cdot V_1$, where the first case corresponds to a particle with x, y, and z axes length equal to 3 nm, 3 nm, and 10 nm. We assume that the dimension of the particle is comparable with the exchange length and then it is expected that the particle is single-domain. This allows one to analyse thermal fluctuations by using the the Fokker-Planck equation. In addition, for the considered geometry, the demagnetizing factors are $N_x = N_y = 0.4523$, $N_z = 0.0954$ and then $k_{\mathrm{eff}} = N_x - N_z = 0.3569$. In the following set of numerical experiments we compared the numerical calculated self-covariance function for magnetization component m_z. The starting state corresponds to the statistical equilibrium [1]. The results are reported in Fig. 1 where a good agreement between the two methods is apparent.

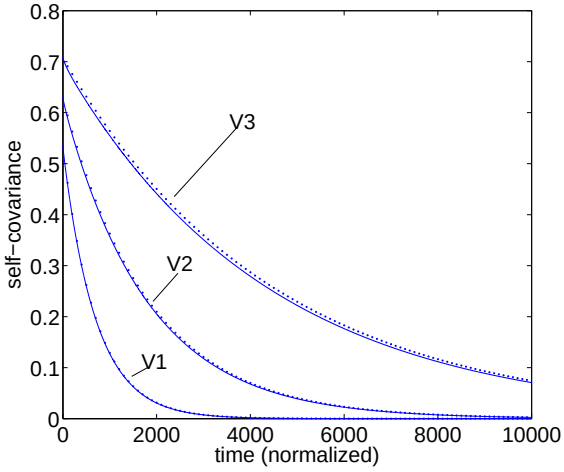

Fig. 1. Comparison between self-covariance functions for magnetization component m_z. Three different volumes are considered $V_1 < V_2 < V_3$. Solid line: self covariance function obtained from the algebraic approach. The number of grid point is $n = 1000$. Dots: computed self-covariance by using the spectral collocation method. The number of grid point is 21.

References

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[3] J. Berrut, L. Trefethen, "Barycentric Lagrange interpolation", *SIAM Review*, vol. 46, pp. 501-517.

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