Relaxation in Chains of Ferromagnetic Crystals

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I. Introduction

A complete description of magnetic hysteresis must include the effect of thermal fluctuations, which cause the magnetism to jump from one state to another before an instability is reached. The relaxation rates depend on the network of connections between stable states by way of saddle points. To determine these connections one must start by finding the saddle points. This is difficult because micromagnetic solvers do not converge on saddle points. Methods such as the nudged elastic band may find a path across a saddle point, but may work poorly if there are a lot of bumps in the energy surface.

A new method is presented that finds all the energy minima and saddle points for a small number of interacting ferromagnets. It is applied to chains of magnetite crystals such as are found in magnetotactic bacteria.

II. Method

Lagrange multipliers are used to express the equilibrium equations as polynomial equations. These are solved using an algorithm that finds all the roots of polynomial systems (*1*). The eigenvalues of the Hessian are used to classify equilibria as minima, saddle points and other. Then a two-stage solver goes downhill from each saddle point to the minimum on each side of the saddle point. Given the matrix of energy barriers between stable states, a master equation is solved and the time dependence of the moment determined.

III. Results

If there are N crystals in the chain with uniaxial anisotropy, there are 4^N solutions to the equilibrium equations. In the limit of no interactions, 2^N of these states are stable (all the combinations of moments up and down along the chain axis). As the interaction strength increases, the number of stable states and saddle points decrease until there are two of each. The relaxation mode for 4 crystals or less is the fanning mode. The mode for 5 crystals or more is a new mode called the two-domain fanning mode (Fig. 1).

In chains of magnetite crystals, the relative strength of interactions is determined by the length-to-width ratio of the particles. As the number of crystals increases, the upper volume limit for superparamagnetism quickly converges on a volume that is almost independent of shape. If the crystals have a weak internal anisotropy, magnetostatic interactions have a strong effect on this critical size even when the particles are a few body lengths apart.

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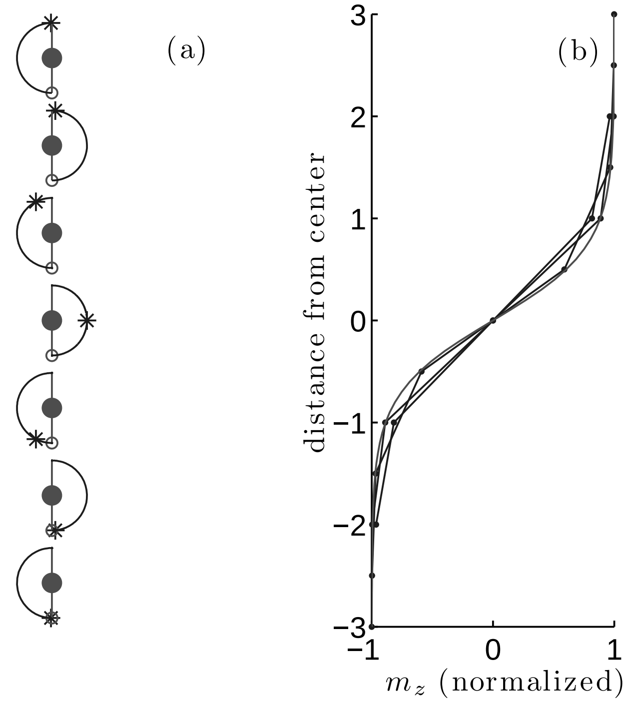

Fig. 1. (a) The two-domain fanning mode for 7 cubic particles in contact with each other, showing the direction of the moments on unit circles. Starting states are open circles and saddle points are stars. From negative saturation to the saddle point, half of the moments rotate most of the way. On the way to positive saturation the other half catch up. (b) Axial components of the moments for chains of $5-7$ particles as a function of distance (in particle lengths) from the center. These fit a tanh function well.

IV. Implications for Micromagnetics

In micromagnetic models, the magnetization is represented by a lattice of unit vectors. These vectors are strongly coupled by the exchange interaction as well as magnetic torques. The results for chains suggest that, in such a system, the number of saddle points may be comparable to the number of stable states. Unfortunately, the method described here is not useful for solving general micromagnetic problems because there would be an astronomical number of complex solutions to the equilibrium equations.

References

[1] J. Verschelde, *ACM Trans. Math. Softw.* **25**, 251 (1999).