

A coupled multipole expansion - finite element approach for dynamic micromagnetic modeling

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I. INTRODUCTION

In numerical micromagnetics, the magnetostatic field evaluation is one of the most time-consuming tasks, mainly when simulating structures that approach the micrometer dimension. Many attempts have been made for its optimization, introducing fast summation algorithms based on fast Fourier transform (FFT), fast multipole method (FMM) [1,2] and FFT on multipoles [3]. In this paper, the attention is focused on the coupling of a multipole expansion technique for magnetostatic term computation to a finite element (FE) approach for effective field handling. The domain is decomposed into 3-D cells. Each cell is discretized into finite elements, whose size is comparable to the exchange length. At every time instant, given a nodal distribution of magnetization, the magnetic scalar potential in a generic node is the sum of two terms. The first term, computed by directly integrating the Green formula, accounts for the contributions due to the nodes belonging to a sufficient number of surrounding cells. The second one derives from a multipole expansion in terms of spherical harmonics and considers the contributions of remaining cells [4]. Once computed the magnetic scalar potential, the effective field is obtained by a nodal FE approach [5]. The instantaneous nodal values of the effective field are then introduced in the Landau-Lifshitz equation, and the magnetization is locally derived from a time-stepping scheme able to preserve the non-convex constraint.

The paper aims at evidencing how the multipole approximation for the magnetostatic field leads to an improvement in terms of CPU time and memory requirement, in respect of a standard Green integration. However, it introduces local approximation, which can affect the instantaneous solution of the magnetostatic field problem and, then, magnetization time evolution. In the full paper, the numerical formulation will be detailed and the influence on accuracy of different parameters (cell number, size of the radius for Green contributions, expansion order) will be analyzed, considering magnetization reversal phenomena in films with different dimensions.

II. NUMERICAL RESULTS

As an example, Fig. 1 reports the spatial distribution of the magnetic scalar potential in a permalloy film ($500 \text{ nm} \times 500 \text{ nm} \times 5 \text{ nm}$), due to a uniform magnetization along the x -direction. The film is decomposed into 10×10 cells. For each cell, the solution is obtained by integrating the exact Green contributions in a limited number of surrounding layers of cells ($N = 3$). It is evident how the multipole expansion additional term significantly corrects the

prediction, leading only to slight discontinuities in correspondence of the cell interfaces.

Starting from the initial state reported in Fig. 1, and assuming the same spatial discretization in finite elements and cells, the magnetization dynamics in the case of a precessional switching is studied. As shown in Fig. 2, the local errors introduced by the multipole approximation weakly propagate in the magnetization time evolution. The proposed procedure leads to a substantial improvement of the numerical efficiency, e.g. CPU time decreases of about four times for each time step, when $N = 3$.

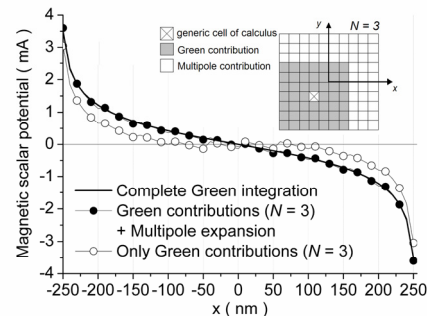


Fig. 1 Spatial distribution of the magnetic scalar potential, computed along x -axis, in a permalloy film ($500 \text{ nm} \times 500 \text{ nm} \times 5 \text{ nm}$) under the hypothesis of a uniform magnetization along the x -direction.

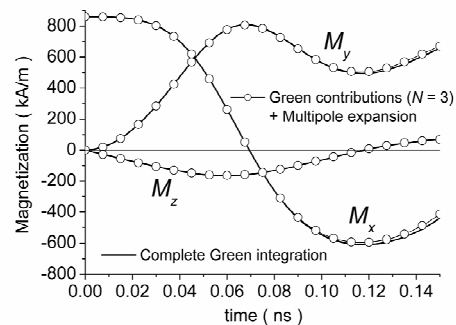


Fig. 2 Time evolution of the average magnetization components in the case of a precessional switching in a permalloy film initially saturated along x -axis and subjected to a uniform external field along y -axis ($H_u = 20 \text{ kA/m}$).

III. REFERENCES

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