Multiscale finite element solution of the exchange term in micromagnetic analysis of large bodies

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I. Introduction

In magnetisation processes upper and lower bounds of spatial scales are determined respectively by the body size, affecting the magnetostatic energy, and by the exchange length, determining the corresponding energy. In large bodies, the numerical solution of Landau-Lifshitz (LL) equation requires a spatial discretisation able to resolve the lowest spatial scale (order of some nm), and simultaneously describe the entire domain under study [1], without leading to unacceptable computational burden. The strategies proposed in literature include theoretical approaches, leading to the definition of a reduced problem that identifies a limiting macroscopic model by the application of Γ convergence theory [2, 3], or numerical techniques (e.g. Fast Multipole Method) to efficiently compute the long range magnetostatic interactions [4]. In this work, we address the problem of numerically computing the exchange field in large bodies by applying the multiscale finite element method (MsFEM) [5].

II. Multiscale finite element formulation

To reduce the second-order derivative in the exchange term, the effective field (H_{eff}) expression in LL equation is derived introducing the following weak form [6]:

$$\int_\Omega H_{eff}\, w\, d\omega = \int_\Omega H_a w\, d\omega + \int_\Omega H_m w\, d\omega + \int_\Omega \frac{2k_{an}}{\mu_0 M_S^2}(\boldsymbol{M}\cdot\boldsymbol{u}_{an})u_{an} w\, d\omega - \int_\Omega \frac{2k_e}{\mu_0 M_S^2}\nabla M\cdot\nabla w\, d\omega \qquad (1)$$

where H_a is the applied field, H_m the magnetostatic field, k_e the exchange costant, $\boldsymbol{M}$ the magnetisation and M_S its saturation value, and w is the finite element shape function.

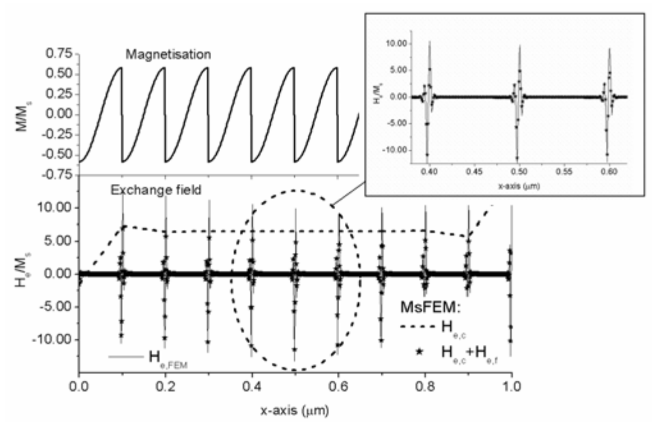

Fig. 1 Spatial distribution of a magnetisation component that simulates a sequence of vortices along the sample (upper plot). The lowest plot shows the reconstruction of the exchange field (enlarged in box).

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A uniaxial anisotropy along $\boldsymbol{u}_{an}$ is assumed (constant k_{an}).

Applying MsFEM, the exchange field H_e is subdivided into a coarse-scale ($H_{e,c}$) and a fine-scale ($H_{e,f}$) component. The first one accounts for the slow spatial variation of magnetisation over the entire domain, while the second one resolves the fast spatial variation related to the exchange length. A coarse mesh subdivides the entire domain (Ω_l) into a given number of sub-domains (Ω_f), with the same fine discretisation. Consequently, the exchange term in (1) is decomposed into two coupled weak forms:

$$\int_{\Omega_f} H_{e,f}\, v\, d\omega = -\frac{2k_e}{\mu_0 M_S^2}\int_{\Omega_f}\nabla M\cdot\nabla v\, d\omega - \int_{\Omega_f} H_{e,l}\, v\, d\omega \qquad (3)$$

$$\int_{\Omega_l} H_{e,c}\, w\, d\omega + \int_{\Omega_l} H_{e,f}\, w\, d\omega = -\frac{2k_e}{\mu_0 M_S^2}\int_{\Omega_l}\nabla M\cdot\nabla w\, d\omega \qquad (4)$$

being v and w the shape functions respectively defined on fine and coarse meshes. Thus, the solution of a large problem is reduced to the solution of a small problem on the coarse mesh (providing $H_{e,l}$) and a sequence of small problems on the fine mesh (providing $H_{e,f}$).

In the example here reported, the magnetisation is assumed to have a spatial dependence simulating a sequence of vortices along one axis (x-axis) of a cubic sample (size 1 μm). Fig. 1 compares the reconstruction of the exchange field using a standard FEM approach ($H_{e,FEM}$), requiring the discretisation of the entire domain, and the MsFEM technique, where the domain has been decomposed into basic cells having size equal to 0.1 μm (coarse mesh). The plots show how the superposition of the coarse-scale and fine scale contributions is able to well reconstruct the effective field. A detailed analysis of the method will be presented at the Conference, also considering the effect of spatial reconstruction on time evolution of magnetization.

III. References

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