Multiscale finite element solution of the exchange term in micromagnetic analysis of large bodies

O. Bottauscio, M. Chiampi, A. Manzin

I. INTRODUCTION

In magnetisation processes upper and lower bounds of spatial scales are determined respectively by the body size, affecting the magnetostatic energy, and by the exchange length, determining the corresponding energy. In large bodies, the numerical solution of Landau-Lifshitz (LL) equation requires a spatial discretisation able to resolve the lowest spatial scale (order of some nm), and simultaneously describe the entire domain under study [1], without leading to unacceptable computational burden. The strategies proposed in literature include theoretical approaches, leading to the definition of a reduced problem that identifies a limiting macroscopic model by the application of convergence theory [2, 3], or numerical techniques (e.g. Fast Multipole Method) to efficiently compute the long range magnetostatic interactions [4]. In this work, we address the problem of numerically computing the exchange field in large bodies by applying the multiscale finite element method (MsFEM) [5].

II. MULTISCALE FINITE ELEMENT FORMULATION

To reduce the second-order derivative in the exchange term, the effective field \( H_{ef} \) expression in LL equation is derived introducing the following weak form [6]:

\[
\int_{\Omega} H_{ef} \cdot w \, d\Omega = \int_{\Omega} H_a \cdot w \, d\Omega + \int_{\Omega} H_m \cdot w \, d\Omega + \int_{\Omega} 2k_e\mu_0 M_S^2 \mu_{an} \cdot w \, d\Omega - \int_{\Omega} 2k_e\mu_0 M_S^2 \nabla M \cdot \nabla \omega \, d\Omega
\]

where \( H_a \) is the applied field, \( H_m \) the magnetostatic field, \( k_e \) the exchange constant, \( M \) the magnetisation and \( M_S \) its saturation value, and \( w \) is the finite element shape function.

A uniaxial anisotropy along \( u_{an} \) is assumed (constant \( k_{an} \)). Applying MsFEM, the exchange field \( H_e \) is subdivided into a coarse-scale \( (H_{e,c}) \) and a fine-scale \( (H_{e,f}) \) component. The first one accounts for the slow spatial variation of magnetisation over the entire domain, while the second one resolves the fast spatial variation related to the exchange length. A coarse mesh subdivides the entire domain \( \Omega_2 \) into a given number of sub-domains \( \Omega_2 \), with the same fine discretisation. Consequently, the exchange term in (1) is decomposed into two coupled weak forms:

\[
\frac{\partial H_{e,c}}{\partial t} = -2k_e\mu_0 M_S^2 \mu_{an} \cdot \nabla M - \frac{\partial}{\partial t} \left( \frac{\partial H_{e,c}}{\partial t} \right) - \frac{\partial}{\partial t} \left( \frac{\partial H_{e,f}}{\partial t} \right) - \frac{\partial}{\partial t} \left( \frac{\partial H_{e,l}}{\partial t} \right)
\]

III. REFERENCES