Representation of memory in particle assembly hysteresis models

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Among the numerous approaches developed to model ferromagnetic hysteresis, two main classes can be envisaged: Preisach models (PM) and particle assembly (PA) models. In both cases the magnetic system is considered as an ensemble of mesoscopic regions or "particles". The Preisach approach, although capable to efficiently describe particle interactions, allows important achievements only if the vector character of the magnetization process is neglected. In PA models, on the other hand, vector properties are properly accounted for, whereas the introduction of interactions is more tricky [1],[2]. Magnetic systems with hysteresis are endowed with non-local memory properties, a feature that makes the $\langle \boldsymbol{M} \rangle$ vs. $\boldsymbol{H}_a$ constitutive law (being $\boldsymbol{H}_a$ and $\langle \boldsymbol{M} \rangle$ the applied field and the sample average magnetization, respectively) depending on past history. One of the most valuable aspects of PM is its ability to track, in the so-called Preisach plane, the field evolution vs. time (t) by means of a line $\boldsymbol{b}(t)$ identifying the state of the system [3]. In order to account for the magnetic history in systems described by PA models (in particular when the interactions are considered: a situation that increases to a large extent the computational complexity), similar approaches have been proposed [2],[4],[5]. Despite to these attempts a general solution is at present still missing; this work is then aimed at giving a contribution to this line of research.

The PA models considered in this paper assume that each mesoscopic region in a magnetic system is characterized by a local anisotropy axis, with anisotropy constant K_u, forming an angle φ_K with a reference direction. A local interaction field $\boldsymbol{H}_i$, collinear with the anisotropy axis (as often assumed [4],[6]) is also present, so that the effective field acting on each particle turns out to be $\boldsymbol{H} = \boldsymbol{H}_a + \boldsymbol{H}_i$. The hysteresis operator adopted is of the Stoner-Wohlfarth type. The behavior of all the particles with the same φ_K is investigated by means of the conventional astroid representation, and exploiting an appropriate frame (different for each φ_K), where the reference direction coincides with the common easy axis. The paths followed with respect to this frame by the effective field $\boldsymbol{H}$ under an alternating or rotating applied field $\boldsymbol{H}_a$ are shown in Fig. 1. For each φ_K a "*memory plane*", with axes $H_K = 2K_u/\mu_0 M_s$ and H_i (being M_s and H_i the saturation magnetization and the modulus of the interaction field, respectively), is introduced. Likewise the PM, each mesoscopic region corresponds to a point of coordinates (H_K, H_i) in this plane. Starting from the energy conditions governing the equilibrium orientation for the local magnetization $\boldsymbol{M}$ (that is the magnetization of a single particle, with modulus $|\boldsymbol{M}| = M_s$), the explicit mathematical expression for a "*metastability curve*", playing the role of the "*metastability cone*" in the Preisach plane [3], is worked out. The applied field history determines the evolution of shape and position of this curve. Consequently, one gets a partition of the memory plane in two simply connected "up" and "down" regions, corresponding to particles with $\cos\vartheta > 0$ and $\cos\vartheta < 0$, respectively, being ϑ the orientation of $\boldsymbol{M}$ with respect to the easy axis (Fig. 1*c*). The analytic expression of the border line $\boldsymbol{b} = \boldsymbol{b}[\varphi_K, H_K; \boldsymbol{H}_a(t)]$, which divides these regions, is obtained. Eventually, considering the whole φ_K domain (-$\pi/2$; $\pi/2$), the procedure outlined above leads to the construction, in the space $[\varphi_K, H_K, H_i]$, of a surface $\{\boldsymbol{b}(t)\}$ storing all the magnetization changes associated with irreversible processes occurred in the system. Due to the fact that any $\boldsymbol{H}_a$ vs. time history can be regarded as a sequence of alternating and rotating episodes, whatever field history can be tracked.

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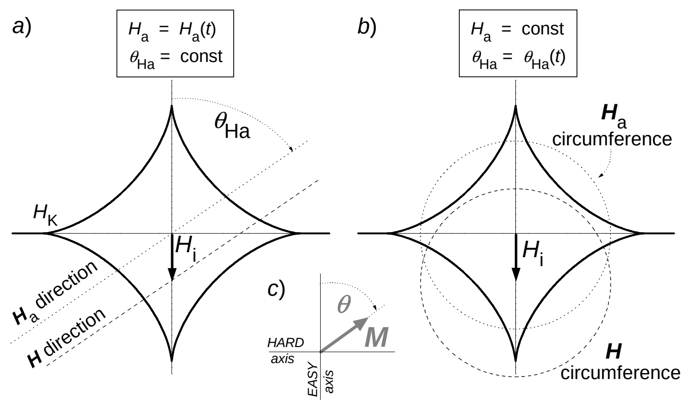

Fig. 1. Paths followed by the effective field $\boldsymbol{H} = \boldsymbol{H}_a + \boldsymbol{H}_i$ (dashed lines) for an alternating *a*) or rotating *b*) applied field $\boldsymbol{H}_a$ (dotted lines), with respect to the astroid corresponding to a particle with easy axis making an angle φ_K with the reference direction. The orientation of the local magnetization $\boldsymbol{M}$ is also sketched *c*).

References

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