Modeling Ferromagnetic Hysteresis

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From measurements on nickel wires and steel pipe we find hysteresis to be described by five superimposed fields [1] as in Eqn.(1). The first is described by differential susceptibilities reflecting exponential increase from a reversal in Eqn. (2) and linear approach to saturation in Eqns. (3) and (4) and shown in the top figure.

$$H_{eff} = H_I + H_D + H_\sigma + H_\rho + H_T \quad (1)$$

$$X_{coop} = X_{rev} e^{(M-M_r)\chi_r / H_c X_i} \quad (2)$$

$$X_{sat} = (M_s - M)\chi_r / H_c \quad (3)$$

$$X_{rev} = X_i \chi_r = X_i (1-m^2)^n \quad (4)$$

Demagnetization factor, D, is defined by the loss of field within a sample with poles and shows a decrease near saturation in the middle figure. H_σ is the Bozorth [2] stress field on magnetization within domains, defined by Eqn. (5), where stress is denoted by σ and magnetostriction is denoted by λ. Wasp-waisted effect of bending nickel wire is shown in both hysteresis loop and differential susceptibility. The eddy current [3] delay field H_ρ is rate of field change times the flux penetration delay time constant, τ. Coercive field increases first linearly and then as the square root of frequency as in Eqn (6). We introduce an indirect thermal field, H_{ind}, defined in Eqn. (7) by change in magnetization due to change in ferromagnetic properties. In Eqn. (8) anhysteretic differential susceptibility is precisely described and shown in the bottom figure.

$$\mu_0 \frac{\partial H_\sigma}{\partial \sigma} = \frac{\partial \lambda}{\partial M} \quad (5)$$

$$H_c \rightarrow H_c + \frac{dH}{dt}\tau \rightarrow H_c + \left[\frac{dH}{dt}\frac{r^2 B_{rem}}{2\rho}\right]^{1/2} \quad (6)$$

$$\frac{dM}{dT} = \frac{dM}{dH}\frac{dH_{ind}}{dT} = \frac{dM}{dX_i}\frac{dX_i}{dT} + \frac{dM}{dH_c}\frac{dH_c}{dT} + \frac{dM}{dM_s}\frac{dM_s}{dT} \quad (7)$$

$$X_{an}^{-1} = D + X_{rev}^{-1} - X_i^{-1} \quad (8)$$

[1] C.S. Schneider, Physica B, **349**, 653-656 (2006)

[2 R.M. Bozorth, Ferromagnetism, (D. Van Nostrand, Princeton, NJ, 1951) p. 546

[3] G. Bertotti, *Hysteresis in Magnetism*; (Academic Press: New York, NY, 1998)

Figure Captions: Top: Saturate hysteresis loop and reversals. Center: Demagnetization field of wires is measured. Bottom: Anhysteretic differential susceptibility is measured and well described.

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