Generalized Wavelet Formulation of Hysteresis Models for Smart Material Systems

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I. INTRODUCTION

The past decade has witnessed the development of *wavelet analysis*, a new tool which emerged from mathematics and was quickly adopted by diverse fields of science and engineering. Wavelet analysis has begun to play a serious role in a broad range of applications, including modeling multiscale phenomena. It calls for a new approach to the use of mathematics in engineering that employs an approach to problems where mathematical process models. Hysteresis phenomena are common forms of nonlinearities that reflect the memory of a system and are associated with many physical phenomena such as ferromagnetism, ferroelectricity, plasticity, superconduction, etc [1]. Currently available predictive algorithms for such nonlinearities are limited in scope. A new approach based on wavelet analysis and Preisach models is to develop a generalized approach to predicting hysteretic efforts for smart material systems. We will address basic research issues relating to the feasibility, methodology and numerical implementation of this new approach.

II. GENERAL SETTINGS

During the last decade, many hysteresis models have been developed and applied to practical problems. An interesting model for ferromagnetism had been proposed by physicist Preisach in 1935 and was extensively used by physicists and engineers because it encompasses the basic feature of the hysteresis phenomena in a conceptually simple and mathematically elegant way. It has been shown that the classical Preisach model can be represented in mathematical form as follows:

$$f(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} u(t) d\alpha d\beta \qquad (1)$$

where u(t) is the input and f(t) is the output of a hysteretic system, $\gamma_{\alpha\beta}$ is the elemental hysteresis operator. The output of each elemental hysteresis operator traces a rectangular loop on the input-output diagram switching from –1 to +1 when the input increases above the threshold α. The output switches from +1 to –1 when the input decrease below the value of β as shown in Figure 1. μ(α,β) is the distribution function which is called Preisach function. The integration takes the past history values of u(t) to determine the current output f(t). The past history is accumulated in the α, β domain by storing the maximum and minimum values of the input variation. If the Preisach function μ(α,β) is known, model (1) can be solved directly by integration. If the function μ(α,β) is not known explicitly, different approach was used to determine the Preisach function.

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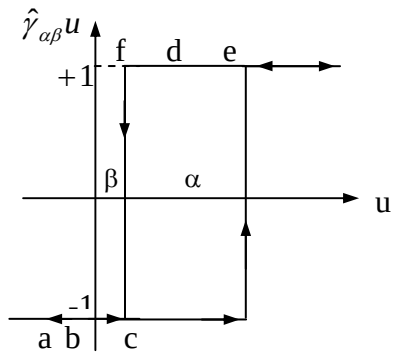

Figure. 1 Elemental hysteresis operator

Preisach modeling can be expressed in integral form as shown in equation (1). Since α, β are the extreme maximum and minimum input series, the system output f(t) in equation (1) can also be considered as a function of variable α, β and denoted by f(α,β). So it can be used to represent the system output.

$$f(\alpha, \beta) = \sum_{k,l} \sum_{p,q=1}^{2} c_{k,l,p,q} \varphi_{k,p}^{j}(\alpha) \varphi_{l,q}^{j}(\beta) \qquad (2)$$

where,

$$\begin{aligned} \varphi_{k,p}^{j}(\alpha) &= 2^{j/2} \varphi_p(2^j \alpha - k) \\ \varphi_{l,q}^{j}(\beta) &= 2^{j/2} \varphi_q(2^j \beta - l) \end{aligned} \qquad (3)$$

It is know that the Preisach function can be written as:

$$\mu(\alpha, \beta) = \frac{1}{2} \frac{\partial^2 f(\alpha, \beta)}{\partial \alpha \partial \beta} \qquad (4)$$

So the Preisach function can be represented by wavelet series which make the calculations and implementations easier.

III. GENERALIZATIONS

Based on the above settings, we will generalize our earlier method in [2] by using different scaling functions and compare the results.

REFERENCES

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[2] Yu, Y. Z. Xiao, E-B. Lin, N. Naganathan, Analytic and Experimental Studies of A Wavelet Identification of Preisach Model of Hysteresis, Journal of Magnetism and Magnetic Materials, 208(3), 255-263, (2000).